

# GAUGE/GRAVITY DUALITY AND SOME APPLICATIONS

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We discuss the AdS/CFT correspondence in which space-time emerges from an interacting theory of D-branes and open strings. These ideas have a historical continuity with QCD which is an interacting theory of quarks and gluons. In particular we review the classic case of D3 branes and the non-conformal D1 brane system. We outline by some illustrative examples the calculations that are enabled in a strongly coupled gauge theory by correspondence with dynamical horizons in semi-classical gravity in one higher dimension. We also discuss implications of the gauge fluid/gravity correspondence for the information paradox of black hole physics.

## 1 Introduction:

A look at the history of elementary particles reveals that a majority of the constituents of the Standard Model, were conceived by theory before they were experimentally established. This includes quarks that were predicted by Murray Gell-Mann[1] and George Zweig[2] to explain hadron spectra. In this note we want to discuss the elementary constituents of space-time and their interactions from which emerges a theory of gravitation.

We hope that the theoretical conceptions we dwell upon in this note contribute to a quantum theory of gravity that enables a description of phenomena like the formation and evaporation process of a black hole and basic issues in cosmology like the origin and fate of the universe. Besides being a framework to address these difficult questions, ‘string theoretic’ methods

seem to have applications to various problems in gauge theories, fluid dynamics and condensed matter physics. This note is written with the hope that the ideas and methods of string theory are accessible to a large community of physicists.

In order to pose the question better we first consider the more familiar setting of quantum chromo-dynamics (QCD) where the quarks interact via color gluons. This theory accounts for the spectrum of hadrons, their interactions and properties of nuclei. In particular in the limit of long wave lengths the chiral non-linear sigma model[3] (another invention of Gell-Mann with Levy) describes the interactions of pions. This theory is characterized by a dimensional coupling, the pion coupling constant  $f_\pi^{-1}$ . In 3+1 dimensions  $f_\pi$  has the dimensions of (mass)<sup>2</sup> and if we generalize the QCD gauge group from  $SU(3)$  to  $SU(N)$ , then  $f_\pi \sim N$ . Hence in the limit of large N the pions are weakly coupled and the theory has soliton solutions with ‘baryon number’. The mass of the baryon is proportional to N, the number of quark constituents<sup>1</sup>. In modern terminology one would say that the chiral model is ‘emergent’ from an underlying theory of more elementary constituents and their interactions. The phenomenon of ‘emergence’ occurs in complex systems in many areas of science, and also social sciences. Gell-Mann has also contributed to this area[6].

*Over the last 25 years one question that has occupied theoretical physicists is: In what sense is gravity an emergent phenomenon? What are the fundamental constituents of ‘space-time’ and their interactions.* The question is quite akin to that of the emergence of the chiral model from QCD. If gravity is the analogue of the chiral model, where the gravitational (Newton) coupling is dimensional and akin to the ‘pion coupling constant’, what is the QCD analogue for gravity, from which gravity is emergent?

Perturbative string theory gave the first hint that gravity may be derived from a more microscopic theory because its spectrum contains a massless spin 2 excitation[7, 8]. However real progress towards answering this question came about with the discovery of D-branes[9].

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<sup>1</sup>This theory, which emerges from QCD, had phenomenological antecedents in the theory of super-conductivity in the work of Nambu and Jona-Lasinio[4, 5].

## 2 D-branes the building blocks of string theory

A D- $p$  brane (in the simplest geometrical configuration) is a domain wall of dimension  $p$ , where  $0 \leq p \leq 9$ . It is characterized by a charge and it couples to a  $(p+1)$  form abelian gauge field  $A^{(p+1)}$ , e.g. a D0 brane couples to a 1-form gauge field  $A_\mu^{(1)}$ , a D1 brane couples to a 2-form gauge field  $A_{\mu\nu}^{(2)}$  etc. The D- $p$  brane has a brane tension  $T_p$  which is its mass per unit volume. The crucial point is that  $T_p \propto 1/g_s$ . This dependence on the coupling constant (instead of  $g_s^{-2}$ ) is peculiar to string theory. It has a very important consequence. A quick estimate of the gravitational field of a D- $p$  brane gives,  $G_N^{(10)} T_p \sim g_s^2/g_s \sim g_s$ . Hence as  $g_s \rightarrow 0$ , the gravitational field goes to zero! If we stack  $N$  D- $p$  branes on top of each other then the gravitational field of the stack  $\sim N g_s$ . A useful limit to study is to hold  $g_s N = \lambda$  fixed, as  $g_s \rightarrow 0$  and  $N \rightarrow \infty$ . In this limit when  $\lambda \gg 1$  the stack of branes can source a solution of supergravity. On the other hand when  $\lambda \ll 1$  there is a better description of the stack of  $D$ -branes in terms of open strings. A stack of  $D$ -branes interacts by the exchange of open strings very much like quarks which interact by the exchange of gluons. Fig. 1a illustrates the self-interaction of a D2-brane by the emission and absorption of an open string and Fig. 1b illustrates the interaction of 2 D2-branes by the exchange of an open string. In the infra-red limit only the lowest mode of the open string contributes and hence the stack of  $N$   $D$ -branes can be equivalently described as a familiar  $SU(N)$  non-abelian gauge theory in  $p+1$  dim.

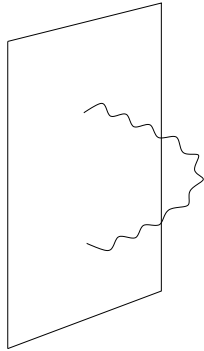


Fig 1a

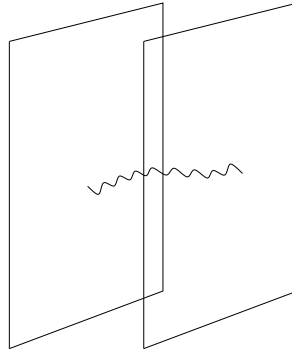


Fig 1b

### 3 Statistical Mechanics of D-brane Systems and Black Hole Thermodynamics

One of the earliest applications of the idea that  $D$ -branes are the basic building blocks of ‘string theory’ (and hence of a theory of gravity) was to account for the entropy and dynamics of certain near extremal black holes in  $4 + 1$  dim. As is well known, Strominger and Vafa[10] in a landmark paper showed that the Benkenstein-Hawking entropy of these black holes is equal to the Boltzmann entropy calculated from the micro-states of a system of D1 and D5 branes

$$S_{BH} = \frac{A_h}{4G_N} = k_B \ell n \Omega = S_{\text{Boltzmann}}$$

This result established that black hole entropy can be obtained from the statistical mechanics of the collective states of the brane system and it provided a macroscopic basis of the first law of thermodynamics,  $dS_{BH} = TdM$ , where  $M$  is the mass of the black hole. Hawking radiation can be accounted for from the averaged scattering amplitude of external particles and the micro-states[11].

### 4 $D3$ branes and the AdS/CFT correspondence

[12, 13, 14, 15, 16]:

We now discuss the dynamics of a large number  $N$ , of D3 branes. A D3 brane is a 3+1 dim. object. A stack of  $N$  D3 branes interacts by the exchange of open strings. In the long wavelength limit ( $\ell_s \rightarrow 0$ ,  $\ell_s$  is the string length), only the massless modes of the open string are relevant. These correspond to 4 gauge fields  $A_\mu$ , 6 scalar fields  $\phi^I$  ( $I = 1, \dots, 6$ ) (corresponding to the fact that the brane extends in 6 transverse dimensions) and their supersymmetric partners. These massless degrees of freedom are described by  $\mathcal{N} = 4$ ,  $SU(N)$  Yang-Mills theory in 3+1 dim. This is a maximally supersymmetric, conformally invariant superconformal field theory in 3+1 dimensions. The coupling constant of this gauge theory  $g_{YM}$ , is simply related to the string coupling  $g_s = g_{YM}^2$ . The ’tHooft coupling is  $\lambda = g_s N$  and the theory admits a systematic expansion in  $1/N$ , for fixed  $\lambda$ . Further as  $\ell_s \rightarrow 0$  the coupling of the D3 branes to gravitons also vanishes, and hence we are left with the  $\mathcal{N} = 4$  SYM theory and free gravitons.

On the other hand when  $\lambda \gg 1$ , various operators of the large  $N$  gauge theory source a supergravity fluctuations in 10-dimension e.g. the energy-momentum tensor  $T_{\mu\nu}$  sources the gravitational field in one higher dimension. The supergravity fields include the metric, two scalars, two 2-form potentials, and a 4-form potential whose field strength  $F_5$  is self-dual and proportional to the volume form of  $S^5$ . The fact that there are  $N$  D3 branes is expressed as  $\int_{S^5} F_5 = N$ . There are also fermionic fields required by supersymmetry. It is instructive to write down the supergravity metric:

$$ds^2 = H^{-1/2}(-dt^2 + d\vec{x} \cdot d\vec{x}) + H^{1/2}(dr^2 + r^2 d\Omega_5^2) \quad (1)$$

$$H = \left(1 + \frac{R^4}{r^4}\right), \quad \left(\frac{R}{\ell_s}\right)^4 = 4\pi g_s N$$

Since  $|g_{00}| = H^{-1/2}$  the energy depends on the 5th coordinate  $r$ . In fact the energy at  $r$  is related to the energy at  $r = \infty$  (where  $g_{00} = 1$ ) by  $E_\infty = \sqrt{|g_{00}|} E_r$ . As  $r \rightarrow 0$  (the near horizon limit),  $E_\infty = \frac{r}{R} E_r$  and this says that  $E_\infty$  is red-shifted as  $r \rightarrow 0$ . We can allow for an arbitrary excitation energy in string units (i.e. arbitrary  $E_r \ell_s$ ) as  $r \rightarrow 0$  and  $\ell_s \rightarrow 0$ , by holding a mass scale ‘ $U$ ’ fixed:

$$\frac{E_\infty}{\ell_s E_r} \sim \frac{r}{\ell_s^2} = U \quad (2)$$

Note that in this limit the gravitons in the asymptotically flat region also decouple from the near horizon region. This is the famous near horizon limit of Maldacena[13] and in this limit the metric (2) becomes

$$ds^2 = \ell_s^2 \left[ \frac{U^2}{\sqrt{4\pi\lambda}} (-dt^2 + d\vec{x} \cdot d\vec{x}) + 4\sqrt{4\pi\lambda} \frac{dU^2}{U^2} + \sqrt{4\pi\lambda} d\Omega_5^2 \right] \quad (3)$$

This is locally the metric of  $\text{AdS}_5 \times S^5$ .  $\text{AdS}_5$  is the anti-de Sitter space in 5 dim. This space has a boundary at  $U \rightarrow \infty$ , which is conformally equivalent to 3+1 dim. Minkowski space-time.

### The AdS/CFT conjecture

*The conjecture of Maldacena is that  $\mathcal{N} = 4$ ,  $SU(N)$  super Yang-Mills theory in 3+1 dim. is dual to type IIB string theory with  $\text{AdS}_5 \times S^5$  boundary conditions.*

The gauge/gravity parameters are related as  $g_{YM}^2 = g_s$  and  $R/\ell_s = (4\pi g_{YM}^2 N)^{1/4}$ . It is natural to consider the  $SU(N)$  gauge theory living on the boundary of  $AdS_5$ . The gauge theory is conformally invariant and its global exact symmetry  $SO(2,4) \times SO(6)$ , is also an isometry of  $AdS_5 \times S^5$ . The metric (3) has a “horizon” at  $U = 0$  where  $g_{tt} = 0$ . It admits an extension to the full  $AdS_5$  geometry which has a globally defined time like killing vector. The boundary of this space is conformal to  $S^3 \times R^1$  and the gauge theory on the boundary is well-defined in the IR since  $S^3$  is compact.

The AdS/CFT conjecture is difficult to test because at  $\lambda \ll 1$  the gauge theory is perturbatively calculable but the dual string theory is defined in  $AdS_5 \times S^5$  with  $R \ll \ell_s$ . On the other hand for  $\lambda \gg 1$ , the gauge theory is strongly coupled and hard to calculate. In this regime  $R \gg \ell_s$  and the string theory can be approximated by supergravity in a derivative expansion in  $\ell_s/R$ . It turns out that for large  $N$  and large  $\lambda$ ,  $D$ -branes source supergravity fields  $\leq$  spin 2. The gravitational coupling is given by

$$G_N \sim g_s^2 \sim \frac{\lambda^2}{N^2} \ll 1$$

Note the analogy with the constituent formula  $f_\pi^{-1} \sim \frac{1}{N}$ . The region  $\lambda \sim 1$  is most intractable as we can study neither the gauge theory nor the string theory in a reliable way. *However since the conjecture can be verified for supersymmetric states on both sides of the duality, one assumes that the duality is true in general and then uses it to derive interesting consequences for both the gauge theory and the dual string theory (which includes quantum gravity).*

#### **Interpretation of the radial direction of AdS:**

Before we discuss the duality further we would like to explain the significance of the extra dimension ‘ $r$ ’. Let us recast the  $AdS_5$  metric by a redefinition:  $\frac{r}{R} = e^{-\phi}$

$$ds^2 = e^{-2\phi} (-dt^2 + d\vec{x} \cdot d\vec{x}) + R^2(d\phi)^2 + R^2 d\Omega_5^2 \quad (4)$$

The boundary in these coordinates is situated at  $\phi = -\infty$ . Now this metric has a scaling symmetry. For  $\alpha > 0$ ,  $\phi \rightarrow \phi + \log \alpha$ ,  $t \rightarrow \alpha t$  and  $\vec{x} \rightarrow \alpha \vec{x}$ , leaves the metric invariant. From this it is clear that the additional dimension ‘ $Re^\phi$ ’ represents a length scale in the boundary space-time:  $\phi \rightarrow -\infty$  corresponds to  $\alpha \rightarrow 0$  which represents a localization or short distances in the boundary coordinates  $(\vec{x}, t)$ , while  $\phi \rightarrow +\infty$  represents long distances on the boundary.

$\phi$  is reminiscent of the Liouville or conformal mode of non-critical string theory, where the idea of the emergence of a space-time dim. from string theory was first seen[17].

The AdS/CFT correspondence clearly indicates that gravity is an emergent phenomenon. What this means is that all gravitational phenomena can be calculated in terms of the correlators of the energy-momentum tensor of the gauge theory, whose microscopic constituents are D-branes interacting via open strings.

## 5 Black holes and AdS/CFT

The  $\mathcal{N} = 4$ , super Yang-Mills theory defined on  $S^3 \times R^1$  can be considered at finite temperature if we work with euclidean time and compactify it to be a circle of radius  $\beta = 1/T$ , where  $T$  is the temperature of the gauge theory. We have to supply boundary conditions which are periodic for bosonic fields and are anti-periodic for fermions. These boundary conditions break the  $\mathcal{N} = 4$  supersymmetry, and the conformal symmetry. However the AdS/CFT conjecture continues to hold and we will discuss the relationship of the thermal gauge theory with the physics of black holes in AdS.

As we have mentioned, in the limit of large  $N$  (i.e.  $G_N \ll 1$ ) and large  $\lambda$  (i.e.  $R \gg \ell_s$ ), the string theory is well approximated by supergravity, and we can imagine considering the Euclidean string theory partition function as a path integral over all metrics which are asymptotic to  $\text{AdS}_5$  space-time. (For the moment we ignore  $S^5$ ).

The saddle points are given by the solutions to Einstein's equations in 5-dim. with a negative cosmological constant

$$R_{ij} + \frac{4}{R^2}g_{ij} = 0 \tag{5}$$

As was found by Hawking and Page, a long time ago, there are only two spherically symmetric metrics which satisfy these equations with  $\text{AdS}_5$  boundary conditions:  $\text{AdS}_5$  itself and a black hole solution.

It was shown in Ref. [15] that the ‘deconfinement’ phase of the gauge theory corresponds to the presence of a large black hole in AdS. The temperature of the black hole is the temperature of the deconfinement phase. The AdS/CFT correspondence says that the equilibrium thermal properties of the gauge theory in the regime when  $\lambda \rightarrow \infty$  are the same as those of

the black hole. This correspondence enables us to make precise quantitative statements about the gauge theory at strong coupling ( $\lambda \gg 1$ ), using the fact that on the AdS side the calculation in gravity is semi-classical.

We list a few exact results of thermodynamics of the gauge theory at strong coupling[15].

- (i) the temperature at which the first order confinement-deconfinement transition occurs:

$$T_c = \frac{3}{2\pi R_{S^3}} \quad (6)$$

where  $R_{S^3}$  is the radius of  $S^3$ .

- (ii) the free energy for  $T > T_c$

$$F(T) = -N^2 \frac{\pi^2}{8} T^4$$

Here we see a typical use of the AdS/CFT correspondence calculations in the strongly coupled gauge theory ( $\lambda \gg 1$ ) which can be done using the correspondence by using semi-classical gravity since  $G_N \sim \frac{1}{N^2} \ll 1$  and  $\frac{R}{\ell_s} \sim \lambda^{1/4} \gg 1$ .

### Conformal Fluid Dynamics and Dynamical Horizons

We have seen that the thermodynamics of the strongly coupled gauge theory in the limit of large  $N$  and large  $\lambda$  is calculable, in the AdS/CFT correspondence, using the thermodynamic properties of a large black hole in  $\text{AdS}_5$  with horizon  $r_h \gg R$ . Similar results hold for a black brane, except that in this case the gauge theory in  $R^3 \times S^1$  is always in the deconfinement phase since  $T_c = 0$  if  $R_{S^3} = \infty$ , by Eq. (6). We now discuss how this correspondence can be generalized to real time dynamics in this gauge theory when both  $N$  and  $\lambda$  are large.

Let us generalize black brane (hole) thermodynamics to fluid dynamics. In conformal fluid dynamics the system is in local thermodynamic equilibrium over a length scale  $L$  so that  $L \gg \frac{1}{T}$ . In the bulk theory in one higher dim. this corresponds to a horizon that is a slowly varying (see Fig. 2) function of the boundary co-ordinates  $(\vec{x}, t)$ .

$$r_h \rightarrow r_h + \delta r_h(\vec{x}, t)$$

$$T \rightarrow T + \delta T(\vec{x}, t)$$



$$\frac{1}{T} \frac{\partial}{\partial x^\mu} \frac{\delta T}{T} \sim \frac{1}{LT} \ll 1$$

The ripples on the horizon of a black brane at the linearized level are analysed in terms of quasi-normal modes with a complex frequencies  $\omega = \omega_R + i\omega_I$ ,  $\omega_I \propto T$ , where  $T$  is the temperature of the non-fluctuating brane. The complex frequency arises because of the presence of a horizon when we impose only ‘in-falling’ boundary conditions. For the dual gauge theory the quasi-normal mode spectrum implies the dissipation of a small disturbance of the fluid in a characteristic time. This is the qualitative reasoning behind the calculation of ‘transport coefficients’ of the gauge theory like viscosity, thermal and heat conductivity which can be done using semi-classical gravity and the Kubo formula for retarded Green’s functions of the corresponding conserved currents. This important step was taken by Policastro, Son and Starinets[18].

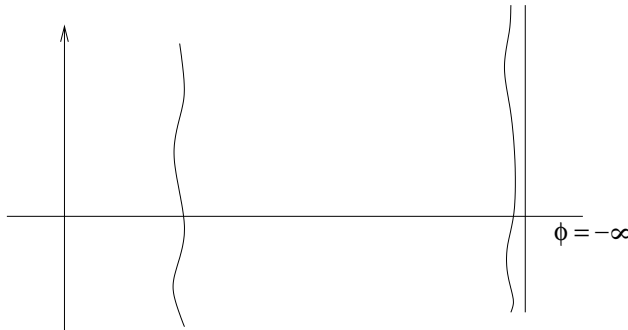


Fig. 2

While linear response theory enables us to calculate transport coefficients of fluid dynamics, we now briefly discuss non-linear fluid dynamics and gravity, and indicate a remarkable connection between the (relativistic) Navier-Stokes equations of fluid dynamics and the long wavelength oscillations of the horizon of a black brane which is described by Einstein’s equations of general relativity with a negative cosmological constant.

On general physical grounds a local quantum field theory at very high density can be approximated by fluid dynamics. In a conformal field theory in  $3 + 1$  dim. we expect the energy density  $\epsilon \propto T^4$ , where  $T$  is the local temperature of the fluid. Hence fluid dynamics is a good approximation for length scales  $L \gg 1/T$ . The dynamical variables of relativistic fluid dynamics are the four velocities:  $u_\mu(x)$  ( $u_\mu u^\mu = -1$ ), and the densities of local

conserved currents. The conserved currents are expressed as local functions of the velocities, charge densities and their derivatives. The equations of motion are given by the conservation laws. An example is the conserved energy-momentum tensor of a charge neutral conformal fluid:

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P\eta^{\mu\nu} - \eta \left( P^{\mu\alpha} P^{\nu\beta} (\partial_\alpha u_\beta + \partial_\beta u_\alpha) - \frac{1}{3} P^{\mu\nu} \partial_\alpha u^\alpha \right) + \dots \quad (7)$$

where  $\epsilon$  is the energy density,  $P$  the pressure,  $\eta$  is the shear viscosity and  $P^{\mu\nu} = u^\mu u^\nu + \eta^{\mu\nu}$ . These are functions of the local temperature. Since the fluid dynamics is conformally invariant (inheriting this property from the parent field theory) we have  $\eta_{\mu\nu} T^{\mu\nu} = 0$  which implies  $\epsilon = 3P$ . Since the speed of sound in the fluid is given by  $v_s^2 = \frac{\partial P}{\partial \epsilon}$ ,  $v_s = \frac{1}{\sqrt{3}}$  or re-instating units  $v_s = \frac{c}{\sqrt{3}}$ , where  $c$  is the speed of light in vacuum. The pressure and the viscosity are then determined in terms of temperature from the microscopic theory. In this case conformal symmetry and the dimensionality of space-time tells us that  $P \sim T^4$  and  $\eta \sim T^3$ . However the numerical coefficients need a microscopic calculation. The Navier-Stokes equations are given by (7) and

$$\partial_\mu T^{\mu\nu} = 0 \quad (8)$$

The conformal field theory of interest to us is a gauge theory and a gauge theory expressed in a fixed gauge or in terms of manifestly gauge invariant variables is not a local theory. In spite of this (7) seems to be a reasonable assumption and the local derivative expansion in (7) can be justified using the AdS/CFT correspondence.

We now briefly indicate that the eqns.(7), (8) can be deduced systematically from black brane dynamics[19]. Einstein's equation (5) admits a boosted black-brane solution

$$ds^2 = -2u_\mu dx^\mu dv - r^2 f(br) u_\mu u_\nu dx^\mu dx^\nu + r^2 P_{\mu\nu} dx^\mu dx^\nu \quad (9)$$

where  $v, r, x^\mu$  are in-going Eddington-Finkelstein coordinates and

$$f(r) = 1 - \frac{1}{r^4} \quad (10)$$

$$u^v = \frac{1}{\sqrt{1 - \beta_i^2}}, \quad u^i = \frac{\beta^i}{\sqrt{1 - \beta_i^2}}$$

where the temperature  $T = 1/\pi b$  and the velocities  $\beta_i$  are all constants. This 4-parameter solution can be obtained from the solution with  $\beta^i = 0$  and  $b = 1$  by a boost and a scale transformation. The key idea is to make  $b$  and  $\beta^i$  slowly varying functions of the brane volume i.e. of the co-ordinates  $x^\mu$ . One can then develop a perturbative non-singular solution of (5) as an expansion in powers of  $1/LT$ . Einstein's equations are satisfied provided the velocities and pressure that characterise (9) satisfy the Navier-Stokes eqns. The pressure  $P$  and viscosity  $\eta$  can be exactly calculated to be [19, 20]

$$P = (\pi T)^4 \text{ and } \eta = 2(\pi T)^3 \quad (11)$$

Using the thermodynamic relation  $dP = s dT$  we get the entropy density to be  $s = 4\pi^4 T^3$  and hence obtain the famous equation of Policastro, Son and Starinets,

$$\frac{\eta}{s} = \frac{1}{4\pi} \quad (12)$$

which is a relation between viscosity of the fluid and the entropy density. Strongly coupled fluid behaves more like a liquid than a gas. Systematic higher order corrections to (7) can also be worked out.

The experiments at RHIC seem to support very rapid thermalization and a strongly coupled quark-gluon plasma with very low viscosity coefficient,  $\frac{\eta}{s} \gtrsim \frac{1}{4\pi}$ .

The fluid dynamics/gravity correspondence can also be used to study non-equilibrium processes like thermalization which are dual to black hole formation in the gravity theory. An important result in this study is that the thermalization time is more rapid than the expected value  $\propto \frac{1}{T}$  where  $T$  is the temperature [21].

Another important result is the connection between the area theorems of general relativity and the positivity of entropy in fluid dynamics [23].

The fluid/gravity correspondence is firmly established for a  $3 + 1$  dim. conformal fluid dynamics which is dual to gravity in  $\text{AdS}_5$  space-time. A similar connection holds for  $2 + 1$  dim. fluids and  $\text{AdS}_4$  space-time. We shall discuss the case of non-conformal fluid dynamics in  $1 + 1$  dim. separately. A special (asymmetric) scaling limit of the relativistic Navier-Stokes equations, where we send  $v_s = \frac{c}{\sqrt{3}} \rightarrow \infty$  leads to the standard non-relativistic Navier-Stokes equations for an incompressible fluid [22]. *In summary we have a truly remarkable relationship between two famous equations of physics viz. Einstein's equations of general relativity and the Navier-Stokes equations.*

Finally it is hoped that the AdS/CFT correspondence lends new insights to the age old problem of turbulence in fluids. Towards this goal the AdS/CFT correspondence has also been established for forced fluids, where the ‘stirring’ term is provided by an external metric and dilaton field[24].

## 6 Non-conformal fluid dynamics in 1+1 dim. from gravity [25, 26]

The famous Policastro, Son and Starinets result (12) is indeed a cornerstone of the gauge/gravity duality. It was originally derived in the context of conformal fluid dynamics. However one suspects that the conjectured bound  $\frac{\eta}{s} \geq \frac{1}{4\pi}$  may be more generally valid. We present a summary of a project of the fluid dynamics description, via the gauge/gravity duality for the case of  $N$  D1 branes at finite temperature  $T$ . The gauge theory describing the collective excitations of this system is a 1+1 dim.  $SU(N)$  gauge theory with 16 supersymmetries. Note that this gauge theory is not conformally invariant. At high temperatures we expect the theory to have a fluid dynamics description, in terms of a 2-velocity  $u^\mu$  and stress tensor

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P\eta^{\mu\nu} - \xi P^{\mu\nu} \partial_\lambda u^\lambda$$

Note that  $\eta_{\mu\nu} T^{\mu\nu} = -3\xi \partial_\lambda u^\lambda$ , where  $\xi$  is the bulk viscosity. The dual gravity description corresponds to 2 regimes. For  $\sqrt{\lambda} N^{-2/3} \ll T \ll \sqrt{\lambda}$ , the gravity dual is a classical solution corresponding to a non-external D1 brane. For  $\sqrt{\lambda} N^{-1} \ll T \ll \sqrt{\lambda} N^{-2/3}$  the gravity solution corresponds to a fundamental string. Here  $\lambda = g_{YM}^2 N$ .

For both regimes we find the following exact answers for the strongly coupled fluid dynamics. There is exactly one gauge invariant quasi-normal mode with dispersion:

$$\omega = \frac{q}{\sqrt{2}} - \frac{i}{8\pi T} q^2 \quad (13)$$

The linearized fluid dynamics equations lead to the dispersion relation:

$$\omega = v_s q - \frac{i\xi}{2(\epsilon + P)} q^2 \quad (14)$$

$v_s^2 = \frac{\partial P}{\partial \epsilon}$  is the velocity of the sound mode. Using  $v_s^2 = \frac{1}{2}$  and the relation

$\epsilon + P = Ts$ , we once more arrive at

$$\frac{\xi}{s} = \frac{1}{4\pi} \quad (15)$$

It is worth pointing out that (15) is valid even if we work with the geometry of D1 branes at cones over Sasaki-Einstein manifolds. Here the corresponding gauge theory is different from the gauge theory with 16 supercharges that we mentioned before. Using similar techniques we have also studied the case of the  $SU(N)$  gauge theory in  $1+1$  dim. with finite  $R$ -charge density. The dual supergravity solution is that of a non-extremal D1 brane spinning along one of the Cartan directions of  $SO(8)$  which reflects the isometry of  $S^7$  present in the near horizon geometry. In this case, besides energy transport, there is also charge transport. The transport coefficients like electrical and heat conductivity can be calculated, and the Weidemann-Franz law can be verified. Once again (14) is valid.

## 7 A New Term in Fluid dynamics:

The fluid dynamics of a charged fluid is described by the conserved stress tensor  $T_{\mu\nu}$  and charged current  $J_\mu$ . The constituent equations are (to leading order in the derivative expansion)

$$T_{\mu\nu} = P(\eta_{\mu\nu} + 4u_\mu u_\nu) - 2\eta\sigma_{\mu\nu} + \dots \quad (16)$$

$$J_\mu = nu_\mu - DP_\mu^\nu D_\nu n. \quad (17)$$

where  $n$  is the charge density. However in the study of the charged black brane dual to a fluid at temperature  $T$  and chemical potential  $\mu$ , a new term was discovered in the charged current

$$J_\mu = nu_\mu - DP_\mu^\nu D_\nu n + \zeta\ell_\mu \quad (18)$$

$\ell_\mu = \epsilon_{\mu\nu\rho\sigma}u_\nu\omega_{\rho\sigma}$ ,  $\omega_{\rho\sigma} = \partial_\rho u_\sigma - \partial_\sigma u_\rho$  (the vorticity). The appearance of the new vorticity induced current in (18) is directly related to the presence of the Chern-Simons term in the Einstein-Maxwell lagrangian in the dual gravity description[27, 28].

In a remarkable paper Son and Surowka[29] showed that the vorticity dependent term in (18) always arises in a relativistic fluid dynamics in which

there is an anomalous axial  $U(1)$  current:  $\partial_\mu J_\mu^A = -\frac{1}{8}CF_{\mu\nu}F_{\rho\sigma}\epsilon_{\mu\nu\rho\sigma}$ . They showed on general thermodynamic grounds that

$$\zeta = C \left( \mu^2 - \frac{2}{3} \frac{\mu^3 n}{\epsilon + P} \right)$$

where  $\epsilon$  and  $P$  are the energy density and pressure, and  $\mu$  is the chemical potential. If  $C = 0$  one recovers the result (17) of Landau and Lifshitz. This new term may be relevant in understanding bubbles of strong parity violation observed at RHIC and generally in the description of rotating charged fluids.

## 8 Implications of the gauge fluid/gravity correspondence for the information paradox of black hole physics

The Navier-Stokes equations imply dissipation and violate time reversal invariance. The scale of this violation is set by  $\eta/\rho$  ( $\eta$  is the viscosity and  $\rho$  is the density) which has the dim. of length (in units where the speed of light  $c = 1$ ). There is no paradox here with the fact that the underlying theory is non-dissipative and time reversal invariant, because we know that the Navier-Stokes equations are not a valid description of the system for length scales  $\ll \eta/\rho$ , where the micro-states should be taken into account. *An immediate important implication of this fact via the AdS/CFT correspondence is that there will always be information loss in a semi-classical treatment of black holes in general relativity.* This fact raises an important question: while we understand that information loss in fluid dynamics because we know the underlying constituent gauge theory, a similar level of understanding does not exist on the string/gravity side, because we as yet do not know the exact equations for all values of the string coupling.

## 9 Concluding remarks:

In this note we have reviewed the emergence, via the AdS/CFT correspondence, of a quantum theory of gravity from an interacting theory of D-branes. Besides giving a precise definition of quantum gravity in terms of non-abelian gauge theory, this correspondence turns out to be a very useful tool to calculate properties of strongly coupled gauge theories using semi-classical gravity.

The correspondence of dynamical horizons and the fluid dynamics limit of the gauge theory enables calculation of transport coefficients like viscosity and conductivity. We also indicated that dissipation in fluid dynamics implies that in semi-classical gravity there will always be ‘information loss’.

We conclude with a brief mention of other applications of the AdS/CFT correspondence to various problems in physics.

### **Condensed Matter:**

The AdS/CFT correspondence offers a tool to explore many questions in strongly coupled condensed matter systems in the vicinity of a quantum critical point. It enables calculation of transport properties, non-fermi liquid behavior, quantum oscillations and properties of fermi-surfaces etc.[30, 31, 32, 33]. There is a puzzling aspect in the application of semi-classical gravity to condensed matter systems: what determines the smallness of the gravitational coupling, which in the gauge theory goes as  $N^{-2}$ ?

Another interesting development is bulk superconductivity i.e. the presence of a charged scalar condensate in a black hole geometry[34]. This has interesting implications for superfluidity in the quantum field theory on the boundary.

### **QCD and Gauge theories:**

The AdS/CFT correspondence is a powerful tool to calculate multi-gluon scattering amplitudes in  $\mathcal{N} = 4$  gauge theories in 3+1 dims. This is done by relating the amplitude to the calculation of polygonal Wilson lines in a momentum space version of  $AdS_5$ . [35, 36]

Even though the basic theory of the quark-gluon plasma is QCD, calculations in  $\mathcal{N} = 4$  gauge theories do indicate qualitative agreement with RHIC observations. We have already remarked that the observed value of  $\frac{\eta}{s}$  in (11) is in qualitative agreement with RHIC data. Another calculation of interest is that of jet quenching which corresponds to computing the drag force exerted by a trailing string attached to a quark on the boundary, in the presence of a AdS black hole[37].

The AdS/CFT correspondence has also yielded a geometric understanding of the phenomenon of chiral symmetry breaking[38, 39].

### **Singularities in Quantum Gravity**

The AdS/CFT correspondence provides a way to discuss the quantum resolution of the singularities of classical general relativity. One strategy

would be to study the resolution of singularities that occur in the gauge theory in the  $N \rightarrow \infty$  limit. Among these are singularities corresponding to transitions of order greater than two[40, 41] which admit a resolution in a double scaling limit. The difficult part here is the construction of the map between the gauge theory singularity and the gravitational singularity. A proposal in the context of the Horowitz-Polchinski cross-over was made in reference 42.

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